AUTRE PROCESSUS :
DIFFUSION MØLLER
PROPAGATEURS DU PHOTON

Diffusion électron électron $e^-e^- \to e^-e^-$

détruire $2e^-$
créer $2e^-$ $\Big\} \to \Big\{ \begin{matrix} \psi\ \psi \\ \bar\psi\ \bar\psi \end{matrix}$

$$\mathcal{H}_{int}(\underset{\sim}{x}) = \frac{1}{2}\frac{q^2}{4\pi}\psi^+(\underset{\sim}{x})\psi(\underset{\sim}{x})\int d^3\vec{x}'\,\frac{\psi^+(t,\vec{x}')\psi(t,\vec{x}')}{|\vec{x}-\vec{x}'|}$$

$$- q\,\bar\psi(\underset{\sim}{x})\,\vec{\gamma}.\vec{A}(\underset{\sim}{x})\,\psi(\underset{\sim}{x})$$

$\to$ contribution de $\mathcal{H}_{Coul}(\underset{\sim}{x})$ au 1er ordre $\propto q^2$

— — — — $q\bar\psi\,\vec\gamma.\vec A\,\psi$ au 2^e ordre $\propto q^2$

<u>Contribution coulombienne</u>

$$S_{fi}^{(c)(1)} = -i\int d^4\underset{\sim}{x} \langle 3,4|\ \frac{1}{2} q^2\,\psi^+(\underset{\sim}{x})\psi(\underset{\sim}{x})\int d^3\vec{x}'\,\frac{\psi^+(t,\vec{x}')\psi(t,\vec{x}')}{4\pi|\vec{x}-\vec{x}'|}\ |1,2\rangle$$

$$= -\frac{i}{2}q^2\sum_{abcd}\langle 3,4|\ b_a^+ b_b\, b_c^+ b_d\ |1,2\rangle \frac{1}{\sqrt{2\omega_a V}}\frac{1}{\sqrt{2\omega_b V}}\frac{1}{\sqrt{2\omega_c V}}\frac{1}{\sqrt{2\omega_d V}}\cdots$$

$$\cdots(u_a^+u_b)(u_c^+u_d)\int d^4\underset{\sim}{x}\,d^4\underset{\sim}{x}'\,e^{i\underset{\sim}{a}.\underset{\sim}{x}}e^{-i\underset{\sim}{b}.\underset{\sim}{x}}\frac{\delta(t-t')}{4\pi|\vec{x}-\vec{x}'|}e^{i\underset{\sim}{c}.\underset{\sim}{x}'}e^{-i\underset{\sim}{d}.\underset{\sim}{x}'}$$

Elément de matrice :

$$\langle\ \ \rangle = \langle 0|b_4 b_3\ b_a^+\,(\delta_{bc} - b_c^+ b_b)\,b_d\ b_1^+ b_2^+|0\rangle$$

$$= \delta_{bc}\langle 0|\, b_4 b_3\, b_a^+\, b_d\, b_1^+ b_2^+|0\rangle \leftarrow \text{nul si } (1,2)\neq(3,4)\ \text{(diffusion)}$$

ou éliminé par : $\mathcal{H}_{int}$:

$$- \langle 0|b_4 b_3\, b_a^+ b_c^+\, b_b\, b_d\, b_1^+ b_2^+|0\rangle$$

(−)

reste :

$$= -(\delta_{a3}\delta_{c4} - \delta_{a4}\delta_{c3})(\delta_{d1}\delta_{b2} - \delta_{b1}\delta_{d2})$$

$$= \delta_{a4}\delta_{c3}(\delta_{d1}\delta_{b2} - \delta_{b1}\delta_{d2}) + \underbrace{\delta_{a3}\delta_{c4}(\delta_{b1}\delta_{d2} - \delta_{d1}\delta_{b2})}$$

même contribution : $\Big\{\begin{matrix} a \leftrightarrow c \\ b \leftrightarrow d\end{matrix} \iff \underset{\sim}{x} \leftrightarrow \underset{\sim}{x}'$

$$S_{fi}^{(c)(1)} = -i\, q^2 \frac{1}{\sqrt{2\omega_4 V}} \frac{1}{\sqrt{2\omega_2 V}} \frac{1}{\sqrt{2\omega_3 V}} \frac{1}{\sqrt{2\omega_1 V}} \cdots$$

$$\cdots \Big\{ (u_4^+ u_2)(u_3^+ u_1) \int d^4\underset{\sim}{x}\, d^4\underset{\sim}{x}'\ e^{i(\underset{\sim}{4}-\underset{\sim}{2})\cdot \underset{\sim}{x}} \frac{\delta(t-t')}{4\pi|\vec{x}-\vec{x}'|}\, e^{i(\underset{\sim}{3}-\underset{\sim}{1})\cdot\underset{\sim}{x}'}$$

$$- (1 \longleftrightarrow 2) \Big\}$$

Représentation intégrale en $\int d^4\underset{\sim}{k}\ e^{-i\underset{\sim}{k}\cdot\underset{\sim}{x}}\ ?$ pour $\frac{\delta(t)}{4\pi|\vec{x}|}$

dimensions $\left[\frac{\delta(t)}{4\pi|\vec{x}|}\right] = L^{-2}$

$[d^4\underset{\sim}{k}] = L^{-4} \implies [\,?\,] = L^2$

~~...~~ paramètres $\underset{\sim}{k}$, $[\underset{\sim}{k}^2] = L^{-2}$, et $\underset{\sim}{x}$ peut-être (?)

mais $\frac{\delta(t)}{4\pi|\vec{x}|}$ pas invariant de Lorentz

mais invariant par rotation $\implies\ ? \propto \frac{1}{\vec{k}^2}$

Essayons :

$$\int d^4\underset{\sim}{k}\ \frac{e^{-i\underset{\sim}{k}\cdot\underset{\sim}{x}}}{\vec{k}^2} = \underbrace{\int dk^0\, e^{-ik^0x^0}}_{=2\pi\,\delta(t)} \int d^3\vec{k}\ \frac{e^{i\vec{k}\cdot\vec{x}}}{\vec{k}^2}$$

reste : $\displaystyle\int d^3\vec{k}\,\frac{e^{i\vec{k}\cdot\vec{x}}}{\vec{k}^2} = \int_0^\infty dk\, \vec{k}^2\frac{1}{\vec{k}^2}\ 2\pi\int_{-1}^{1} d(-\cos\theta)\ e^{-i|\vec{k}||\vec{x}|(-\cos\theta)}$

$$= 2\pi\int_0^\infty dk\ \frac{e^{-ikx}-e^{-ikx}}{-ikx} = \frac{2i\pi}{x}\, 2\int_0^\infty dk\ \underbrace{\frac{-i\sin kx}{k}}_{\text{paire}}$$

$$= -\frac{2i\pi}{x}\int_{-\infty}^{\infty} dk\ \frac{e^{ikx}}{k} = -\frac{2i\pi}{x}\int_0^\pi i\,d\theta = \frac{2\pi^2}{x}$$

$$\implies \int d^4\underset{\sim}{k}\ \frac{e^{-i\underset{\sim}{k}\cdot\underset{\sim}{x}}}{\vec{k}^2} = 4\pi^3\,\frac{\delta(t)}{x} = (2\pi)^4\,\frac{\delta(t)}{4\pi x}$$

$$\implies \frac{\delta(t-t')}{4\pi|\vec{x}-\vec{x}'|} = \int \frac{d^4\underset{\sim}{k}}{(2\pi)^4}\ \frac{e^{-i\underset{\sim}{k}\cdot(\underset{\sim}{x}-\underset{\sim}{x}')}}{\vec{k}^2}$$

$$S_{fi}^{(c)(1)} = -i\,q^2\,\frac{1}{\sqrt{V}}\frac{1}{\sqrt{V}}\frac{1}{\sqrt{V}}\frac{1}{\sqrt{V}}\Big\{(u_4^+u_2)(u_3^+u_1)\cdots$$

$$\cdots\int d^4\underset{\sim}{x}\, d^4\underset{\sim}{x}'\,\frac{d^4\underset{\sim}{k}}{(2\pi)^4}\ e^{i(\underset{\sim}{4}-\underset{\sim}{2})\underset{\sim}{x}}\ \frac{e^{-i\underset{\sim}{k}\cdot(\underset{\sim}{x}-\underset{\sim}{x}')}}{\vec{k}^2}\ e^{i(\underset{\sim}{3}-\underset{\sim}{1})\underset{\sim}{x}'} - (1\Leftrightarrow 2)\Big\}$$

$$S_{fi}^{(c)(4)} = i\,(2\pi)^4\,\delta^4\left(\underset{\sim}{4}+\underset{\sim}{3}-\underset{\sim}{1}-\underset{\sim}{2}\right)\frac{1}{\sqrt{2\omega_1 V}}\frac{1}{\sqrt{\ }}\frac{1}{\sqrt{\ }}\frac{1}{\sqrt{\ }}\cdots$$

$$\cdots\left\{\left(\bar u_4\,(-i)q\,\gamma^0 u_2\right)\frac{1}{\vec k^2}\left(\bar u_3\,(-i)q\,\gamma^0 u_1\right) - (1 \leftrightarrow 2)\right\}$$

$$\underset{\sim}{k} = \underset{\sim}{1}-\underset{\sim}{3} = \underset{\sim}{4}-\underset{\sim}{2} \qquad \underset{\sim}{k} = \underset{\sim}{2}-\underset{\sim}{3} = \underset{\sim}{4}-\underset{\sim}{1}$$

~~1.45~~

Contribution de photon transverse

$$S_{fi}^{(T)(2)} = \frac{(-i)^2}{2}\int d^4\underset{\sim}{x}\,d^4\underset{\sim}{x}'\,\langle 34|\,T\left\{\bar\psi(\underset{\sim}{x})(-q\vec\gamma.\vec A(\underset{\sim}{x}))\psi(\underset{\sim}{x})\;\bar\psi(\underset{\sim}{x}')(-q\vec\gamma.\vec A(\underset{\sim}{x}'))\psi(\underset{\sim}{x}')\right\}|12\rangle$$

$$= \frac{1}{2}\int d^4\underset{\sim}{x}\,d^4\underset{\sim}{x}'\,\langle 34|:\bar\psi(\underset{\sim}{x})\,iq\,\vec\gamma.\underbrace{\vec A(\underset{\sim}{x})\,\psi(\underset{\sim}{x})\,\bar\psi(\underset{\sim}{x}')\,iq\,\vec\gamma.\vec A(\underset{\sim}{x}')}\,\psi(\underset{\sim}{x}'):|12\rangle$$

{on peut calculer ce propagateur pour le champ de Proca,
{mais dénominateur m_A $\Rightarrow$ difficulté des théories de jauge ($m_A=0$)

$$= \frac{1}{2}\sum_{abcd}\langle 34|:b_a^+ b_b\, b_c^+ b_d:|12\rangle\frac{1}{\sqrt{2\omega_a V}}\frac{1}{\sqrt{2\omega_b V}}\frac{1}{\sqrt{2\omega_c V}}\frac{1}{\sqrt{2\omega_d V}}\cdots$$

$$\cdots(\bar u_a iq\gamma^i u_b)(\bar u_c iq\gamma^j u_d)\int d^4\underset{\sim}{x}\,d^4\underset{\sim}{x}'\;e^{i(\underset{\sim}{a}-\underset{\sim}{b}).\underset{\sim}{x}}\,\underbrace{A_i(\underset{\sim}{x})\,A_j(\underset{\sim}{x}')}\,e^{i(\underset{\sim}{c}-\underset{\sim}{d}).\underset{\sim}{x}'}$$

$$\left\{\langle\quad\rangle = \delta_{a4}\,\delta_{c3}\,(\delta_{d1}\delta_{b2}-\delta_{d2}\delta_{b1}) + \left\{\begin{matrix} a\leftrightarrow c\\ b\leftrightarrow d\end{matrix}\right\}\right.$$

$$= \frac{1}{\sqrt{2\omega_1 V}}\frac{1}{\sqrt{2\omega_2 V}}\frac{1}{\sqrt{2\omega_3 V}}\frac{1}{\sqrt{2\omega_4 V}}\cdots$$

$$\cdots\left\{(\bar u_4\,iq\gamma^i u_2)(\bar u_3\,iq\gamma^j u_1)\int d^4\underset{\sim}{x}\,d^4\underset{\sim}{x}'\;e^{i(\underset{\sim}{4}-\underset{\sim}{2}).\underset{\sim}{x}}\,\underbrace{A_i(\underset{\sim}{x})A_j(\underset{\sim}{x}')}\,e^{i(\underset{\sim}{3}-\underset{\sim}{1}).\underset{\sim}{x}'}\right.$$

$$\left. - (1\leftrightarrow 2)\right\}$$

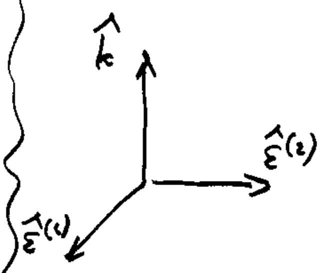
$$\underbrace{A_i(\underset{\sim}{x})\,A_j(\underset{\sim}{x}')} = \begin{cases} A_i(\underset{\sim}{x})A_j(\underset{\sim}{x}') - :A_i(\underset{\sim}{x})A_j(\underset{\sim}{x}'): & t>t' \\ A_j(\underset{\sim}{x}')A_i(\underset{\sim}{x}) - :A_i(\underset{\sim}{x})A_j(\underset{\sim}{x}'): & t'>t \end{cases}$$

$$= \begin{cases} \left[A_i^{(+)}(\underset{\sim}{x}),\ A_j^{(-)}(\underset{\sim}{x}')\right] \\ \left[A_j^{(+)}(\underset{\sim}{x}'),\ A_i^{(-)}(\underset{\sim}{x})\right] \end{cases}$$

choix de base de polar rectiligne ε réels

$$= \frac{1}{\mathcal{V}} \sum_{\vec{k}} \frac{e^{\mp i \underset{\sim}{k}\cdot(\underset{\sim}{x}-\underset{\sim}{x}')}}{2\omega} \sum_{\tau} \left(\hat{\varepsilon}^{(\tau)}_{\vec{k}}\right)_i \left(\hat{\varepsilon}^{(\tau)}_{\vec{k}}\right)_j$$

à mode $\vec{k}$ donné, $\sum_{\tau=1}^{2} \left(\hat{\varepsilon}^{(\tau)}_{\vec{k}}\right)_i \left(\hat{\varepsilon}^{(\tau)}_{\vec{k}}\right)_j = \ ?$



Choix de trièdre de projection:
$\hat{\varepsilon}^{(1)}$, $\hat{\varepsilon}^{(2)}$ et $\hat{k} = \hat{\varepsilon}^{(3)}$

Alors $\left(\hat{\varepsilon}^{(r)}\right)_i = \delta_{ir}$

et $\sum_{r=1}^{3} \left(\hat{\varepsilon}^{(r)}\right)_i \left(\hat{\varepsilon}^{(r)}\right)_j = \delta_{ij}$

$$\Rightarrow \sum_{r=1}^{2} \left(\hat{\varepsilon}^{(r)}\right)_i \left(\hat{\varepsilon}^{(r)}\right)_j = \delta_{ij} - \left(\hat{\varepsilon}^{(3)}\right)_i \left(\hat{\varepsilon}^{(3)}\right)_j$$

$$= \delta_{ij} - \frac{k_i k_j}{\vec{k}^2}$$

et c'est une fonction paire de $\vec{k}$.

$$\underbrace{A_i(\underset{\sim}{x})\,A_j(\underset{\sim}{x}')} = \frac{1}{V}\sum_{\vec{k}}\left(\delta_{ij} - \frac{k_i k_j}{\vec{k}^2}\right) e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}\,\frac{e^{\mp i\omega(t-t')}}{2\omega}$$

$$\sim \frac{1}{(2\pi)^3}\int d^3\vec{k}\left(\delta_{ij} - \frac{k_i k_j}{\vec{k}^2}\right) e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}\,\frac{i}{2\pi}\int dk^0\,\frac{e^{-ik^0(t-t')}}{\underset{\sim}{k}^2+i\varepsilon}$$

$$\boxed{\underbrace{A_i(\underset{\sim}{x})A_j(\underset{\sim}{x}')} = i\int\frac{d^4\underset{\sim}{k}}{(2\pi)^4}\,\frac{\delta_{ij} - \frac{k_i k_j}{\vec{k}^2}}{\underset{\sim}{k}^2+i\varepsilon}\,e^{-i\underset{\sim}{k}(\underset{\sim}{x}-\underset{\sim}{x}')}}$$

$\overbrace{\text{paire en } \underset{\sim}{k}} \Longrightarrow$ signe indifférent)

intégration sur $\int d^4\underset{\sim}{x}' \longrightarrow (2\pi)^4\,\delta^4(\underset{\sim}{k}+\underset{\sim}{3}-\underset{\sim}{1})$

$\int\frac{d^4\vec{k}}{(2\pi)^4} \longrightarrow \qquad \underset{\sim}{k} = \underset{\sim}{1}-\underset{\sim}{3}$

$\int d^4\underset{\sim}{x} \longrightarrow \qquad (2\pi)^4\,\delta^4(\underset{\sim}{4}-\underset{\sim}{2}-\underset{\sim}{k})$

$$S_{fi}^{(T)(2)} = i(2\pi)^4\,\delta^4(\underset{\sim}{4}+\underset{\sim}{3}-\underset{\sim}{1}-\underset{\sim}{2})\,\frac{1}{\sqrt{2\omega_1 V}}\,\frac{1}{V}\,\frac{1}{V}\,\frac{1}{V}\cdots$$

$$\cdots\left\{(\bar{u}_4\, iq\gamma^i u_2)\,\frac{\delta_{ij} - \frac{k_i k_j}{\vec{k}^2}}{\underset{\sim}{k}^2+i\varepsilon}\,(\bar{u}_3\, iq\gamma^j u_1) - (1\leftrightarrow 2)\right\}$$

$\underset{\sim}{k} = \underset{\sim}{1}-\underset{\sim}{3}$
$= \underset{\sim}{4}-\underset{\sim}{2}$

Récapitulation : 4 contributions $\begin{cases} \text{Coulomb : directe, échange} \\ \gamma \text{ transv : " " } \end{cases}$

$$\begin{cases} -i\mathcal{M}^{(c)}_{fi\ \text{dir.}} = \left[\bar{u}_4(-iq\gamma^0)u_2\right]\dfrac{i}{\vec{k}^2}\left[\bar{u}_3(-iq\gamma^0)u_1\right] \\ -i\mathcal{M}^{(T)}_{fi\ \text{dir.}} = \left[\bar{u}_4\, iq\gamma^i\, u_2\right] i\,\dfrac{\delta_{ij}-\frac{k_ik_j}{\vec{k}^2}}{\underset{\sim}{k}^2+i\varepsilon}\left[\bar{u}_3\, iq\gamma^j\, u_1\right] \end{cases}$$

Total :

$$-i\mathcal{M}_{fi\ \text{dir.}} = \left[\bar{u}_4(-iq\gamma^\mu)u_2\right] D_{\mu\nu}(\underset{\sim}{k})\left[\bar{u}_3(-iq\gamma^\nu)u_1\right]$$

avec

$$\begin{cases} D_{00}(\underset{\sim}{k}) \hat{=} \dfrac{i}{\vec{k}^2} \\ D_{ij}(\underset{\sim}{k}) \hat{=} i\,\dfrac{\delta_{ij}-\frac{k_ik_j}{\vec{k}^2}}{\underset{\sim}{k}^2} \\ \text{autres composantes } \hat{=} 0 \end{cases}$$

Ça n'a rien d'évidemment invariant,
et pourtant ....

Propagateur du photon

Pour ~~mettre~~ tenter de mettre $D_{\mu\nu}$ sous une forme tensorielle, faire apparaître le dénominateur $\underset{\sim}{k}^2$ (déjà dans D_{ij}) comme pour un propagateur de champ scalaire.

$$\frac{1}{\vec{k}^2} = \frac{\underset{\sim}{k}^2}{\vec{k}^2}\,\frac{1}{\underset{\sim}{k}^2} = \frac{k^{0\,2}-\vec{k}^2}{\vec{k}^2}\,\frac{1}{\underset{\sim}{k}^2} = -\frac{1}{\underset{\sim}{k}^2}+\frac{k^{0\,2}}{\vec{k}^2}\,\frac{1}{\underset{\sim}{k}^2}$$

Sensationnel :

$$\begin{cases} D_{00}(\underset{\sim}{k}) = -i\,\dfrac{\eta_{00}}{\underset{\sim}{k}^2} + \dfrac{i}{\vec{k}^2\,\underset{\sim}{k}^2}\,k_0k_0 \\ D_{ij}(\underset{\sim}{k}) = -i\,\dfrac{\eta_{ij}}{\underset{\sim}{k}^2} - \dfrac{i}{\vec{k}^2\,\underset{\sim}{k}^2}\,k_ik_j \\ D_{0i} = D_{i0} = 0 = -i\,\dfrac{\eta_{i0}}{\underset{\sim}{k}^2} \end{cases}$$

$$-i\mathcal{M}_{fi\ dir} = \left[\bar{u}_4 (-i)q\gamma^\mu u_2\right] \frac{-i\eta_{\mu\nu}}{\underset{\sim}{k}^2} \left[\bar{u}_3 (-i)q\gamma^\nu u_1\right]$$

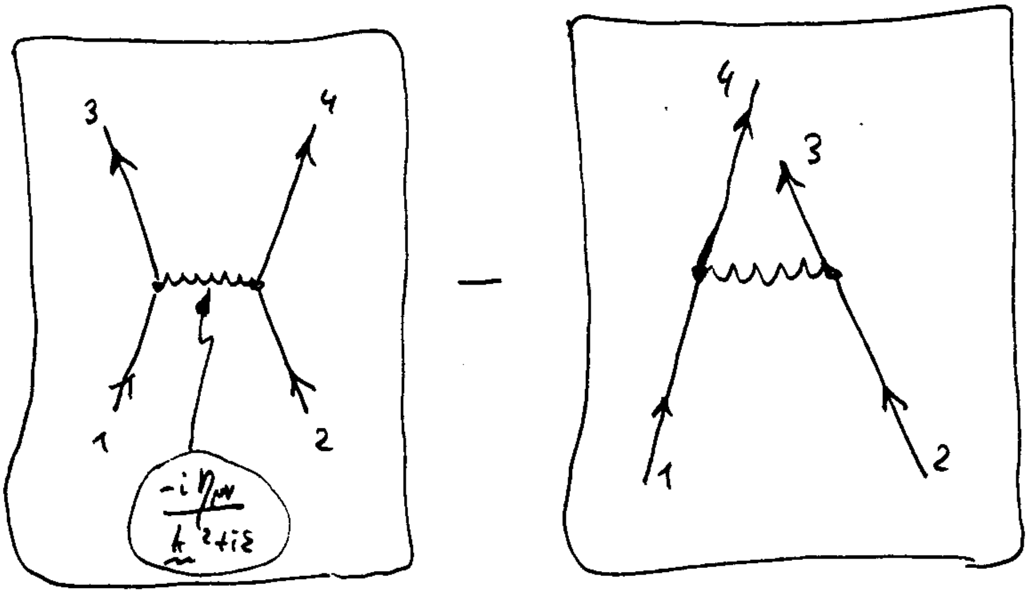
$$-i\frac{q^2}{k^2 \underset{\sim}{k}^2} \left\{ (\bar{u}_4 \gamma^0 u_2)\, k_0 k_0\, (\bar{u}_3 \gamma^0 u_1) - (\bar{u}_4 \gamma^i u_2)\, k_i k_j\, (\bar{u}_3 \gamma^j u_1) \right\}$$

$$\begin{cases} (\not p - m)u = 0 \\ \bar{u}(\not p - m) = 0 \end{cases} \quad \text{avec} \quad \underset{\sim}{k} = \underset{\sim}{p_1} - \underset{\sim}{p_3} = \underset{\sim}{p_4} - \underset{\sim}{p_2}$$

spineurs de base solutions de l'éq. de Dirac $(\not p - m)u = 0$

$$\begin{cases} \bar{u}_4 \gamma^0 k_0 u_2 = \bar{u}_4 \gamma^0 (p_{40} - p_{20}) u_2 \\ \quad = \bar{u}_4 (-\gamma^i p_{4i} + m) u_2 - \bar{u}_4 (-\gamma^i p_{2i} + m) u_2 \\ \quad = -(\bar{u}_4 \gamma^i u_2) k_i \\ \bar{u}_3 \gamma^0 k_0 u_1 = \bar{u}_3 \gamma^0 (p_{10} - p_{30}) u_1 \\ \quad = -\bar{u}_3 (-\gamma^j p_{3j} + m) u_1 + \bar{u}_3 (-\gamma^j p_{1j} + m) u_1 \\ \quad = -(\bar{u}_3 \gamma^j u_1) k_j \end{cases}$$

$\longrightarrow$ ne reste que la contribution tensorielle

$-i\mathcal{M}_{fi} =$ 

$$-i\mathcal{M}_{fi} = (\bar{u}_4 (-i)q\gamma^\mu u_2) \frac{-i\eta_{\mu\nu}}{\underset{\sim}{k}^2} (\bar{u}_3 (-i)q\gamma^\nu u_1) - (\bar{u}_4 (-i)q\gamma^\mu u_1) \frac{-i\eta_{\mu\nu}}{\underset{\sim}{k}'^2} (\bar{u}_3 (-i)q\gamma^\nu u_2)$$

Remarquer:

$$\cancel{(\bar{u}_4 \not{k} u_2)(\bar{u}_3 \not{k} u_1) = [\bar{u}_4 (\not{p}_4 - \not{p}_2) u_2][\bar{u}_3 (}$$

$$\begin{aligned} \bar{u}_4 \not{k} u_2 &= \bar{u}_4 (\not{p}_4 - \not{p}_2) u_2 \\ &= \bar{u}_4 (m - m) u_2 \\ &= 0 \end{aligned}$$

de même $\quad \bar{u}_3 \not{k} u_1 = 0$

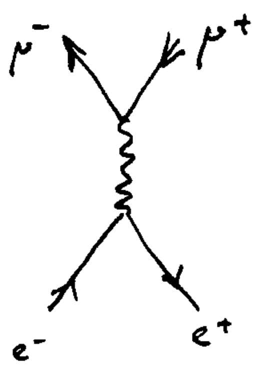
$\Longrightarrow$ on peut aussi bien prendre, pour le propagateur du photon

$$\boxed{-i \, \frac{\eta_{\mu\nu} + \lambda \frac{k_\mu k_\nu}{\underset{\sim}{k}^2}}{\underset{\sim}{k}^2 + i\varepsilon}}$$

avec λ quelconque ; indétermination liée à l'invariance de jauge. Choix le plus commode selon le calcul que l'on a à faire; pour nous $\lambda = 0$ (graphes en arbre, ordre le + bas)

On a maintenant tout ce qu'il faut pour calculer les processus électrodynamiques à l'ordre le + bas (arbres ; pas de boucles)

p. ex. $\quad e^- e^+ \longrightarrow \mu^- \mu^+$ $\qquad$ (cf. notes 1981-82) pour un calcul complet de section efficace avec sommation sur les polar.



~~1.15~~

2.00

The end